REPORT NO. 710/466

ARMOR PLATE

An Analysis of Firings of Cal. .50 A.P. Ammunition Against Homogeneous Armor Plate

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November 26, 1942

WATERTOWN ARSENAL
WATERTOWN, MASS.

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Report No. 710/466
Watertown Arsenal
Problem J1

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OBJECT

To represent the data in Watertown Arsenal Report No. 710/456 by concise formulae.

CONCLUSIONS

1. The data in W.A. Report No. 710/456 on the Army ballistic limits of homogeneous plates against cal. .50 A.P. ammunition are represented very well, up to angles of obliquity of 30°, by the formula

$$V_A = V_1 \ (e/d)^{0.735} + C(1-\cos\theta).$$

Here V_1 is the army ballistic limit at normal incidence against matching plate, e is the plate thickness, d is the core calibre. The constant C, as well as V_1, is a function only of the plate hardness.

2. The corresponding data for the Navy ballistic limits are represented fairly well by the formula

$$V_N = V_2 \ (e/d)^{0.5} \cos^{-2}\theta.$$

Here V_2 is the Navy ballistic limit at normal incidence against matching plate. It is a function only of plate hardness.

3. The relation between the Army and the Navy ballistic limits given in W.A. Report 710/456 is represented, for normal incidence, very well by the following formula.

$$V_N^2 - V_A^2 = V_3^2.$$

The constant V_3 is a function only of plate hardness. It is the velocity the core must have, at the instant of penetration, just to perforate the plate.

4. The energy a calibre 0.50 bullet must have to perforate completely a homogeneous plate in the thickness range 3/8" to 1" varies directly with the plate thickness, all other factors being held constant.

5. The rate at which the navy ballistic limit with respect to calibre 0.50 projectiles increases with angle of obliquity is much higher than can be accounted for by an effective increase in length of projectile path. The data examined in this report suggests the possibility that this rapid rise with obliquity is associated with the projectile breaking up at angles of attack of 20° and over.

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Introduction

Many ballistic formulae have been proposed[1]. Previous attempts to reduce a compilation of data to a concise formula have been motivated primarily by a desire to obtain a convenient measure of the quality of armor plate. Thus suppose that the ballistic limit V of a plate with respect to a standard projectile varies in a known manner with thickness of plate e and with angle of obliquity, θ. Such a formula may be written as

$$V = K\ f\ (e,\theta),$$

Here the factor K is independent of e and θ, but varies with different plate qualities. Then the effectiveness of a given plate in stopping the standard projectile is completely specified by the associated value of K, high values denoting good quality, low values denoting poor quality.

Ballistic formulae may be of use in the development of new types of bullets. A change in a bullet, such as a change in shape of ogive, may improve its penetration ability in certain circumstances, while effecting it adversely in others. Such complicated effects may best be summarized as changes in parameters of the function f.

Finally, ballistic limit formulae are invaluable to an understanding of the mechanism of armor penetration. Any detailed theory of penetration must lead to a ballistic formula. The validity of a particular theory may be tested therefore by a comparison of the corresponding ballistic formula with that which best fits the experimental data.

In the construction of a ballistic formula it is necessary to have firing records of plates of different thicknesses examined at different obliquities. The more uniform the quality of the plates, the more significance can be attached to the derived formula. Little significance may in fact be attached to formulae derived from data upon production plates covering a wide range of thicknesses, for the hardness values of such plates decreases with increasing plate thickness. Other qualities not manifested in the hardness value may also be different in plates of different thicknesses. Sullivan has recently published a report[2] which contains data ideally suited to form the basis for a ballistic formula. The plates of all thicknesses were of the same nominal composition.

Plates of each thickness were so heat treated as to contain specimens covering a wide hardness range. The ballistic limits were obtained using the criterion of complete perforation (navy limit), and also using the criterion that a small pinhole be formed (army limit). The ammunition, calibre 0.50 A.P., was fired at 0°, 20°, 30°, 40° obliquity. All these data are reduced in the present report to concise formulae, except that pertaining to 40° obliquity.

Experimental Errors in Data

An analysis of any set of experimental data should start with an examination of the magnitude of the uncertainties in the data. Only by this means can one decide how closely a derived formula can be expected to fit the experimental data.

The data upon which the present analysis is based consist of the ballistic limits of plates of a given Brinell hardness and of a given thickness, taken at a certain angle. For a given ammunition the ballistic limit is regarded as a function only of these three variables: hardness, thickness, obliquity. Uncertainties in the ballistic limit arise both from errors in measurement, and from the presence of differences in plates which are not reflected in their measured and reported hardness values. The measured value of the ballistic limit is commonly regarded as differing from the true value by not more than ± 25 ft/sec. This limit is obtained by bracketing the ballistic limit by two shots differing by not more than 50 ft/sec.

The data in reference 2 present a convenient method of predicting the uncertainty of the individual values of ballistic limits associated with a given hardness value, thickness and obliquity. Of the many plates tested, six may be grouped into pairs having nearly identical average Brinell hardness: 302 and 304, 329 and 331, 361 and 363.

Nineteen pairs of ballistic limits were determined on these plates, including both the Army and the Navy limits at the various angles of obliquity. The difference between each pair of ballistic limits is recorded as Fig. 1, the difference being recorded as positive if the harder of the two plates has a higher limit. From this figure it may be seen that the distribution of the differences is symmetrical about the origin. The maximum discrepancy is 180 ft./sec.. The mean of the magnitude of these differences is 66 ft./sec.. Such a quantity is known technically as the mean absolute deviation, z.

In order to obtain from this mean absolute deviation the probable error of a single datum it is necessary to assume some sort of error function. This will be taken as the Gaussian error function. In this case the standard deviation is given by[3] $\sqrt{\pi/2}\ z$. Now the standard deviation of a single datum is $(1/\sqrt{2})$ times the standard deviation of the difference of two data of the same quantity. Finally, with a Gaussian distribution of errors, the probable error[3] of a datum is 0.674·(the standard deviation). One thus obtains that there is a 50-50 chance that the ballistic limit of a plate of given hardness, thickness, and obliquity, lies within $\pm\ 0.674 \times (2)^{-1/2} (\pi/2)^{1/2} \times 66$ ft/sec, namely - 39 ft/sec, of a single determination.

Reorganization of Data

Sullivan's data is given in Appendix A. These data give the ballistic limits for plates of various thicknesses, 3/8" to 1", at various angles of obliquity. From 5 to 9 plates of each thickness were tested, the average hardness of these plates varying between 241 and 415 Brinell. Since the hardness of a given plate is in general different from the hardness of all other plates, an individual ballistic limit of a plate can only in rare cases justly be compared with the ballistic limit of a plate of another thickness. It therefore seemed best to confine attention to three hardness ranges, and to analyze only the ballistic limits averaged over each range. In choosing these ranges a compromise had to be made between the two conflicting desires to include as many points as possible, and to make the range as narrow as possible. The ranges chosen were 260-282, 300-310 and 360-390 Brinell. An average was taken of the observed ballistic limits in each range for the various plate thicknesses and angles of obliquity. The averages are given in Table I. In those few cases where no observations lay within a given hardness range, an estimate was made from the observations lying on either side of the range by using Sullivans smooth curves.

Derivation of Ballistic Limit Formulae

A. Army ballistic limit formula.

A plot on log paper of the Army ballistic limits for

TABLE I

Ballistic Limits averaged over Hardness Ranges.
(Cal. .50 A.P. ammunition)

	Army Ballistic Limit				Navy Ballistic Limit			
Angle	0°	20°	30°	40°	0°	20°	30°	40°
Plate Thickness				260-282 B.H.				
3/8"	-	-	1260	1760	1450	1740	2000	2400
1/2"	1293	1488	1922	2427	1759	2192	2397	2427
5/8"	1460*	1850*	2170*	2590*	1420*	2400*	2670*	-
3/4"	1770	2093	2375	-	2159	2370	2542	-
1"	2195	2354	-	-	2463	2647	-	-
				300-310 B.H.				
3/8"	-	-	1420	1900	1480	1870	2070	2550
1/2"	1339	1521	2259	2602	1840	2247	2287	2726
5/8"	1523	1870	2442	2729	1996	2176	2442	2757
3/4"	1851	2243	2610	-	2246	2636	2867	-
1"	2269	2647	-	-	2548	2756	-	-
				360-380 B.H.				
3/8"	-	-	1750	2130	1500	2060	2200	2700
1/2"	1375*	1900*	2440*	2800*	1660*	2010*	2460*	2840*
5/8"	1643*	2240*	2730*	-	2120*	2400*	2730*	-
3/4"	1905	2566	2951	-	2292	2724	3001	-
1"	2473	2853	-	-	2731	2907	-	-

*estimated average

normal incidence is given as Fig.2. Three parallel straight lines may be drawn through the observations, one for each of the three hardness ranges. The slope of these lines is 0.785. The ballistic limit therefore varies as $e^{0.785}$. The associated proportionality constant is most readily found by writing the equation for the Army ballistic limit at zero obliquity in the form

$$V_A = V_1 \ (e/d)^{0.785} , \qquad (1)$$

where d is the calibre of the projectile core, and e is the plate thickness. The constant V_1 has dimensions of velocity. It is equal to the Army ballistic limit at zero obliquity for matching plate ($e = d$). The constant V_1 associated with each hardness range is therefore equal to the ordinate of the corresponding straight line at $e = d$. In this way the values 1130, 1180, 1210 are found for the three hardness ranges, respectively.

An attempt was made to modify Eq.(1), so as to take account of obliquity, by multiplying the right hand member by some function of the obliquity angle θ. If obliquity could be taken care of by such a multiplicative function, then the plot on log paper of ballistic limit against plate thickness would have the same slope for all angles of obliquity. This was found not to be the case.

An attempt was next made to take account of obliquity by adding to the right member of Eq.(1) some function of θ. If this is to be successful, a plot of ballistic limit against angle will give identical curves aside from a vertical shift, for all plates in a given hardness range. Such a plot is given as Fig. 3. Since the curves for all plate thicknesses are essentially parallel, we are justified in taking account of obliquity by adding to the right member of Eq.(1) some function of θ. A successful analytical function can be obtained only by trial. The function C $(1-\cos\theta)$ has been found to be satisfactory up to 40° for the softer plates, up to 30° for the harder plates. A comparison with observations is given as Fig.4, where the increase in ballistic limits due to obliquity is plotted against $(1-\cos\theta)$. If this function is correct, the observations should lie upon straight lines passing through the origin. The constant C associated with a given hardness range is equal to the slope of the corresponding line.

The final Army ballistic limit formula is therefore

$$V_A = V_1 \, (e/d)^{0.785} + C\,(1-\cos\theta). \qquad (2)$$

The coefficients V_1 and C are given as functions of plate hardness in Fig. 5.

B. Navy ballistic formula.

A plot of all the navy ballistic limits in Table I is given on log paper as Fig.6. In nearly every case, the observations for a given hardness range at a given angle of obliquity scatter symmetrically about a straight line with a slope of 1/2. The navy ballistic limit therefore varies with plate thickness approximately as follows:

$$V_N \propto e^{0.50} \qquad (3)$$

The fact that the observations for 0°, 20° and 30° obliquity all lie upon parallel lines means that the effect of obliquity may be taken care of in Eq.(3) by means of a multiplicative factor, thus

$$V_N = V_2\,(e/d)^{0.50}\,f(\theta), \qquad (4)$$

where f is some, at present undetermined, function of obliquity angle θ. f shall arbitrarily be set equal to unity at normal obliquity. Then V_2 is equal to the navy ballistic limit of matching plate at normal incidence. The value of f at 20° and at 30° then gives the ratio of the Navy ballistic limits at these angles to that at zero obliquity. The experimental data are reproduced fairly well by setting f equal to 1.15 and 1.30 for 20° and for 30°, respectively, in all three hardness ranges. These values are nearly reproduced by the analytical function $\cos^{-2}\theta$. A comparison is given in Table II.

Table II
Obliquity Factor

	0	10°	20°	30°
$f(\theta)$	1	-	1.15	1.30
$\cos^{-2}\theta$	1	1.03	1.12	1.33

Eq.(4) is represented by full lines in Fig. 6, with $f(\theta)$ given by Table II, and with V_2 adjusted for each hardness range to give the best fit. These values are 1600, 1600, 1700 ft./sec. for the ranges 260-280, 300-310 and 360-400, respectively. Most of the deviations in Fig. 6 of the observed ballistic limits from the straight lines associated with Eq.(4) show no consistent trend in the three hardness ranges. They may therefore be attributed to the uncertainties in the measurements. Two deviations are consistent in all three hardness ranges. Namely, the observed ballistic limits of the thinnest plates, 3/8", at normal obliquity, and of the thickest plates, 1", at 20° obliquity, are lower than they should be according to Eq.(4). These deviations must be considered as real, and are discussed in the following section.

Theoretical Interpretations

According to Eq.(4), the velocity required for a cal. .50 projectile just to perforate a plate varies as the square root of the plate thickness. The corresponding energy therefore varies linearly with the plate thickness. This linear relation has both interesting theoretical and practical implications.

Theoretical considerations show that if the core perforates the plate by pushing aside the plate material with no net forward motion, then the energy for perforation varies linearly with the plate thickness[4]; while if the plate material is pushed only forward, the energy for perforation varies quadratically with the plate thickness[1]. According to Eq. (4), the plates investigated by Sullivan were therefore perforated essentially by the plate material being pushed aside. At normal incidence, Eq. (4) may be written as

$$\tfrac{1}{2}\, m\, V_N^{\,2} = \left\{ \tfrac{1}{2}\, m\, V_1^{\,2}/d^3 \right\} e\, d^2, \qquad (5)$$

where m is the mass of the core. The bracketed factor in the right hand member of this equation has dimensions of pressure. According to reference 1, this factor should have the same order of magnitude as the tensile strength. Taking V_1 as 1600 ft/sec, we find that this factor is 350,000 psi.

It has been reported that the energy lost by a projectile in passing through a plate is independent of the striking velocity of the projectile[5]. According to this conclusion, and to Eq. (5), a cal. .50 projectile will require the same energy to perforate a plate of given thickness as to perforate two plates, each plate being of half this thickness, provided the latter plates are at least 3/8" thick. Previous reports[6] from this arsenal have concluded that it took a greater total thickness of a plate to defeat a cal. .50 projectile if the plate were in two sections rather than in one. Such a conclusion was based upon experiments using plates considerably harder, of about 440 Brinell, than those used in the data reviewed in this report. The equivalence of two separate 1/2" plates to one 1"plate is based upon the supposition that the projectile strikes the second plate under the same conditions as it strikes the first plate. However, when the projectile strikes the second plate the core has been stripped of its jacket. The penetrative ability of a cal. .50 projectile is considerably reduced by the removal of its jacket[7], presumably because the jacket tends to prevent the nose of the core from breaking up[7]. It is therefore anticipated that two 3/8" plates will be more effective than one 3/4" plate, or two 1/2" plates more effective than a single 1" plate at all angles of obliquity, provided all the plates are homogeneous with a hardness in the range 260 - 400 Brinell, and provided they are separated by a distance at least equal to the length of the projectile.

The rapidity with which the ballistic limit rises with obliquity is very surprising. If the only advantage of obliquity were to lengthen the path of the projectile through the armor plate, the energy necessary for perforation would vary with obliquity at most as $1/\cos\theta$, the navy ballistic limit therefore would vary at most as $1/\cos^{0.5}\theta$. This would make the navy ballistic limit at 20° obliquity 3% higher than at 0° obliquity. From Eq. (4) and Table II, we see that actually it is 15% higher, a rise of five times that predicted by the above naive interpretation. The increase of effective mean length of path can therefore be only a minor factor in the rapid rise of ballistic limit with obliquity. A possible explanation is that the cores of the cal. .50 bullets break up upon passing through the plates at angles of 20° and over. The conditions under which these cores break up has not been reported in the literature.*

The observed navy ballistic limits in all three hardness ranges for 3/8" at normal impact are lower than those given by Eq. (4). It is known[1] that as the thickness of a plate decreases, penetration involves less pushing the plate material aside, more pushing the plate material forward. This change in type of penetration will result[1] in the energy of perforation falling below that given by the formula (4). It is possible that at a thickness of 3/8" this effect becomes apparent for cal. .50 bullets.

The observed navy ballistic limits in all three hardness ranges of 1" plates at 20° obliquity fall below the values given by Eq. (4). This discrepancy may arise because such a comparatively large amount of energy is necessary to perforate a 1" plate at normal incidence that the breaking up of the bullet core at obliquities can not increase the energy for perforation by such a

*Note added Dec. 15. After reading the first draft of this report, Mr. MacDonough of the Watertown Arsenal Armor Plate Range has made a brief survey of the conditions under which the cores of cal. .50 AP cores fracture. He found that all cores which perforated 3/8" homogeneous plates at 20° obliquity were fractured, over the range of incident velocities used, from 1600 to 2900 ft/sec.

large factor as for thinner plates.

It is to be expected that at normal incidence the difference between the energy necessary just to perforate a plate and just to penetrate a plate will be independent of the plate thickness. In order to test this relation, a plot was made, given as Fig.7, in which V_N^2 is compared with V_A^2. Their difference is seen to be essentially independent of plate thickness, that is,

$$V_N^2 - V_A^2 = V_3^2, \qquad (6)$$

where V_3 depends only upon the plate hardness. The constant V_3 is just the velocity the core must have at complete penetration in order just to perforate the plate. The dependence of V_3 upon hardness is given in Fig. 5.

Eqs. (2), (4) and (6) are not consistent with one another. This inconsistency in no way invalidates their usefullness in representing the data fairly well over the limited range of the ratio e/d associated with the data of Table I, namely over the range of e/d from 0.87 to 2.3.

References

1. C. Zener and H. Hollomon: "Mechanism of Armor Penetration, First Partial Report", Watertown Arsenal Report No. 710/454, September, 1942.

2. J. Sullivan: "Ballistic Properties of Rolled Face Hardened Armor and Rolled Homogeneous Armor of Various Hardnesses at Normal Incidence and at Various Obliquities", Watertown Arsenal Report No. 710/456, September 1942.

3. Whitaker and Robinson: "Calculus of Observations" Chapter 8 (Blackie, 1929).

4. H. A. Bethe: "Attempt of A Theory of Armor Penetration" Ordnance Laboratory, Frankford Arsenal, 1941.

5. G. Reynolds, R. Kramer, and W. Bleakney: "Ballistic Tests of Small Arms Plates for the Frankford Arsenal," N.D.R.C. Report No. A-67, July 1942.

6. E. L. Reed and S. L. Kruegel; "Test of Laminated Thin Armor Plate", Watertown Arsenal Report No. 710/275, December 1938.

7. C. M. Hudson: "Development of caliber .60 and 20-15 mm. Ammunition", Ordnance Laboratory, Frankford Arsenal, Report R-157.

Appendix A

Sullivan's Summary of Ballistic Results

(Cal. .50 Projectile)

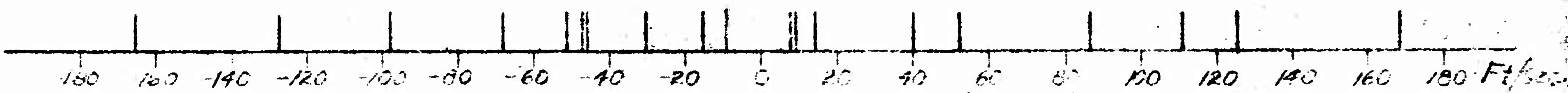
Ballistic Limits

		Army						Navy					
Gage	BHN	0°	10°	20°	30°	40°	45°	0°	10°	20°	30°	40°	45°
3/8"	241	969	-	1048	1130	1575	2120	1464	-	1642	1994	2095	2406
3/8"	245	-	-	-	1165	1732	2203	1407	-	1646	1880	2405	2585
3/8"	329	-	-	-	1507	-	2405	1522	-	-	2135	-	2629
3/8"	331	-	-	-	1618	1919	2530	1472	-	2182	2126	2627	2715
3/8"	341	-	-	-	1315	2113	2491	1511	-	1899	2027	2645	2801
3/8"	415	-	-	-	1970	2248	2521	1500	-	2165	2260	2639	2899
1/2"	261	1267	-	1488	1922	2427	-	1748	-	2192	2397	2427	-
1/2"	282	1320	-	-	-	-	-	1770	-	-	-	-	-
1/2"	302	1339	-	1521	2259	2602	-	1840	-	2247	2287	2726	-
1/2"	321	1415	-	1860	2402	2796	-	1901	-	2206	2402	2796	-
1/2"	415	1350	-	1902	2470	2800	-	1522	-	1902	2636	2883	-
5/8"	255	1439	-	1838	2046	2530	-	1896	-	2510	2785	-	-
5/8"	302	1523	-	1870	2442	2729	-	1996	-	2176	2442	2757	-
5/8"	359	1646	-	2161	2688	-	-	2177	-	2161	2688	-	-
5/8"	409	1639	-	2355	2773	-	-	2024	-	2355	2799	-	-
5/8"	415	1640	-	2219	2881	-	-	1999	-	2464	2881	-	-
3/4"	269	1798	1786	2010	2415	-	-	2185	2174	2438	2513	-	-
3/4"	271	1742	1739	2176	2336	-	-	2130	2164	2303	2572	-	-
3/4"	302	1825	1907	2269	2601	-	-	2242	2293	2552	2863	-	-
3/4"	304	1877	1809	2218	2618	-	-	2250	2264	2721	2870	-	-
3/4"	363	1921	1959	2451	2932	-	-	2316	2316	2638	2932	-	-
3/4"	378	1924	2133	2682	3025	-	-	2272	2412	2851	3070	-	-
3/4"	388	1886	2005	2570	2897	-	-	2318	2341	2792	-	-	-
3/4"	388	1890	2367	2560	-	-	-	2263	2394	2617	-	-	-

Appendix A (Cont'd)

Ballistic Limits

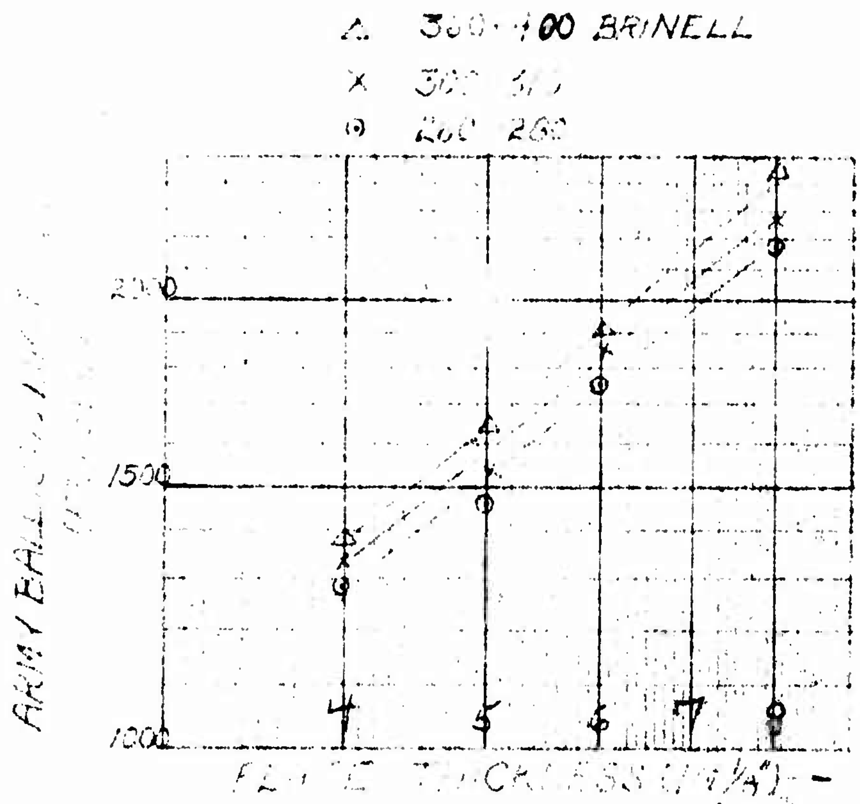
Gage	BHN	Army 0°	Army 10°	Army 20°	Army 30°	Army 40°	Army 45°	Navy 0°	Navy 10°	Navy 20°	Navy 30°	Navy 40°	Navy 45°
1"	263	2208	2256	-	-	-	-	2471	2505	-	-	-	-
1"	272	2205	2265	2354	-	-	-	2475	2734	2647	-	-	-
1"	279	2173	-	-	-	-	-	2444	-	-	-	-	-
1"	304	2269	2287	2647	-	-	-	2548	2620	2756	-	-	-
1"	361	2509	2465	2868	-	-	-	2705	2742	2956	-	-	-
1"	363	2441	2505	2703	-	-	-	2698	2696	2829	-	-	-
1"	368	2480	2481	2869	-	-	-	2713	2872	2906	-	-	-
1"	370	2486	2577	2893	-	-	-	2711	2687	2913	-	-	-
1"	387	2451	2577	2932	-	-	-	2728	2772	2932	-	-	-



The distance of each vertical line from the origin gives the difference in the ballistic limits of a pair of plates of nearly the same BHN, measured at a definite obliquity. The vertical line is placed to the right of the origin if the harder plate of the pair had the higher ballistic limit, to the left if it had the lower ballistic limit.

FIG. 1

ILLUSTRATION OF UNCERTAINTY OF MEASUREMENTS



ARMY BALL[illegible]
ZERO OBLIQUITY

FIG 2

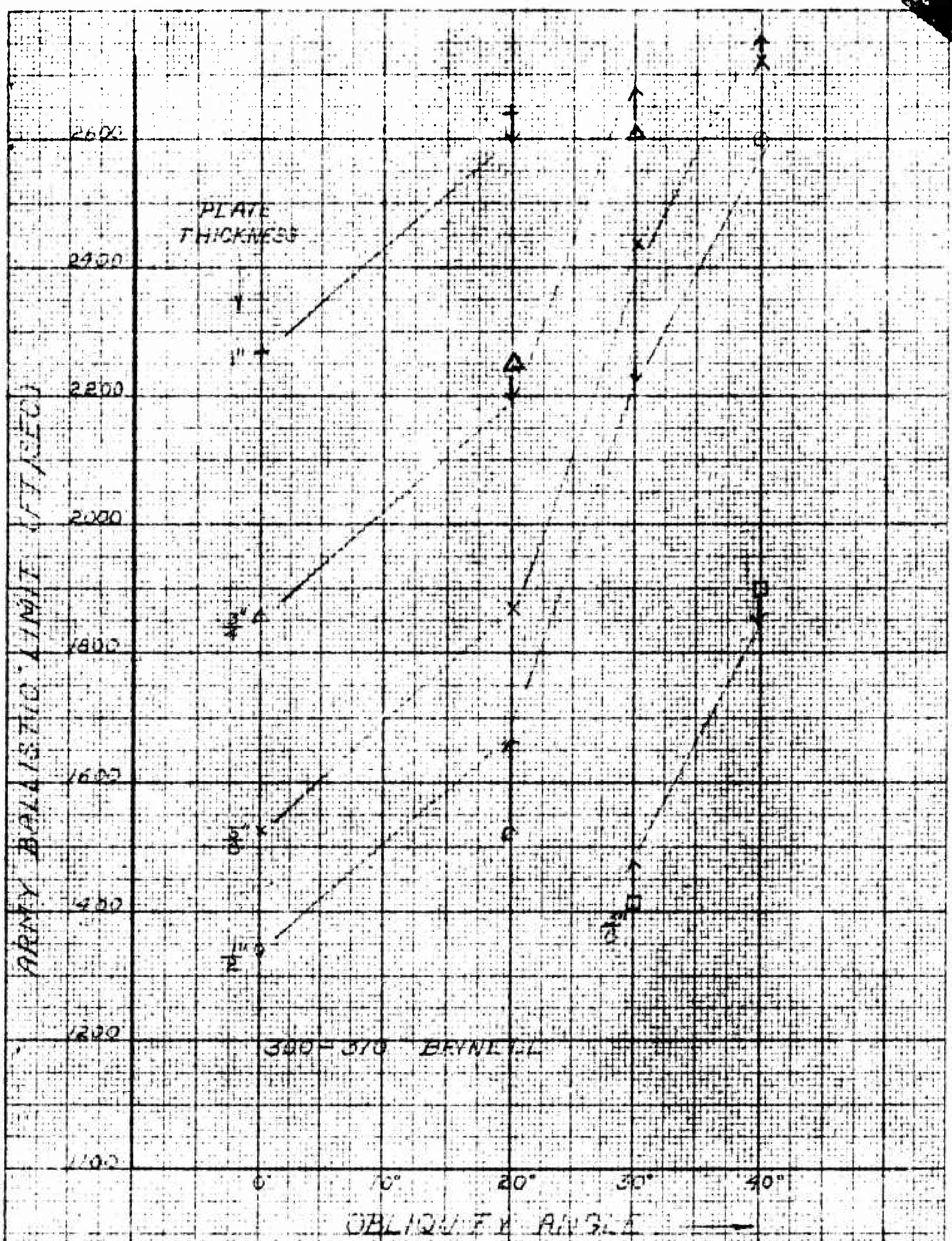
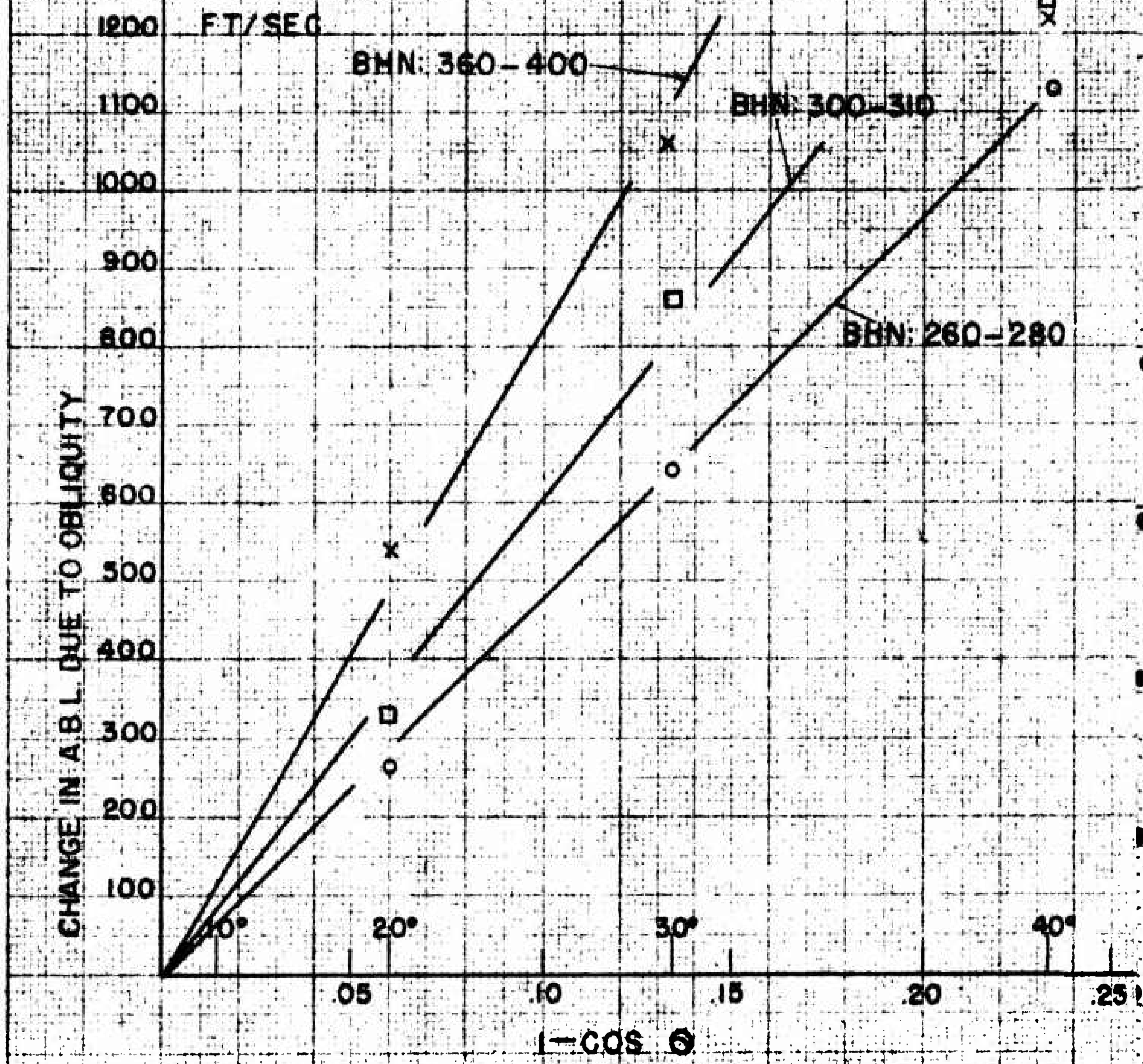
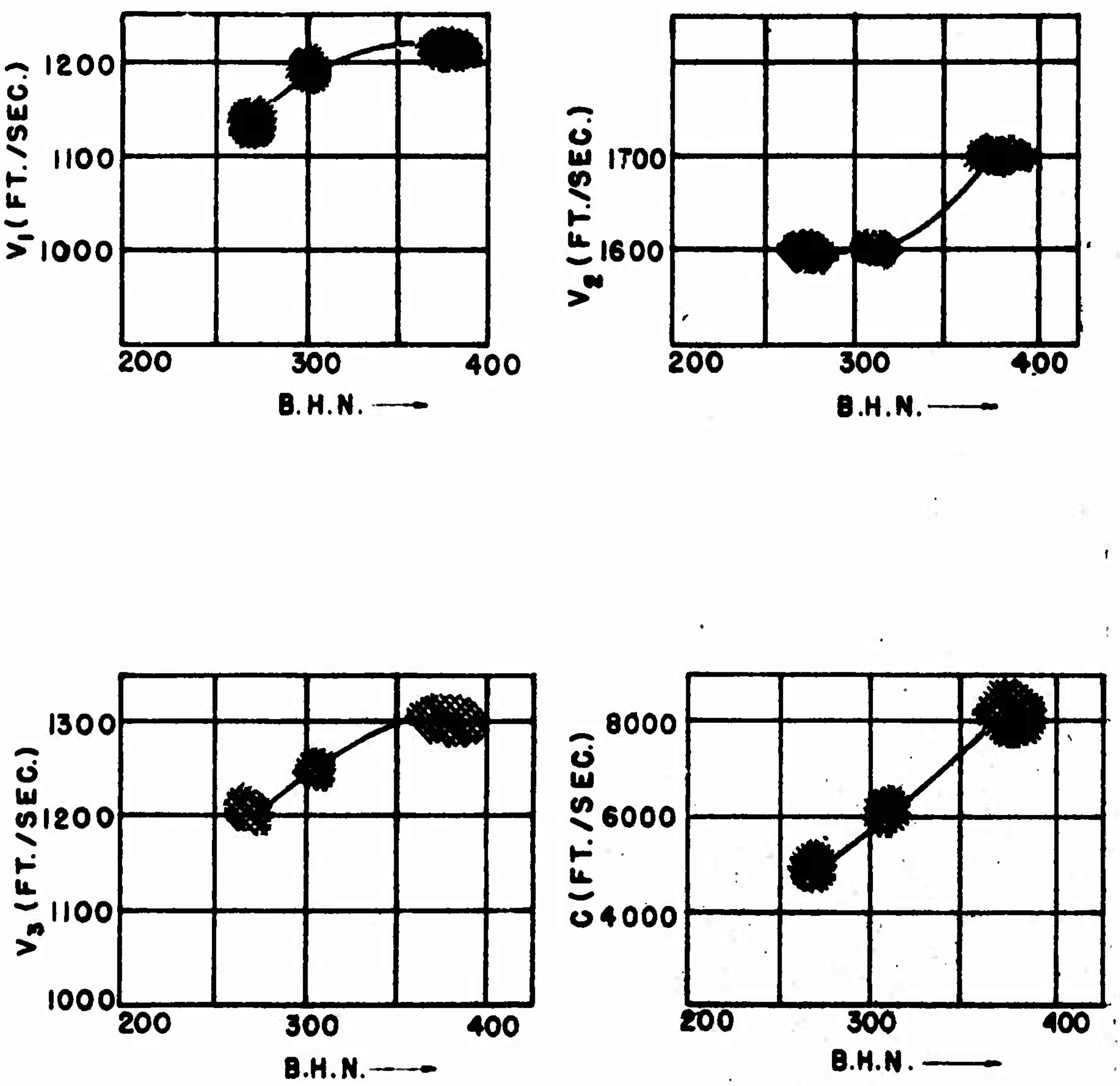


PLATE THICKNESS
ARMY BALLISTIC LIMIT (FT/SEC)
2600
2400
2200
2000
1800
1600
1400
1200
1100
1"
3/4"
5/8"
1/2"
300-370 BRINELL
0°
10°
20°
30°
40°
OBLIQUITY ANGLE
FIG. 5



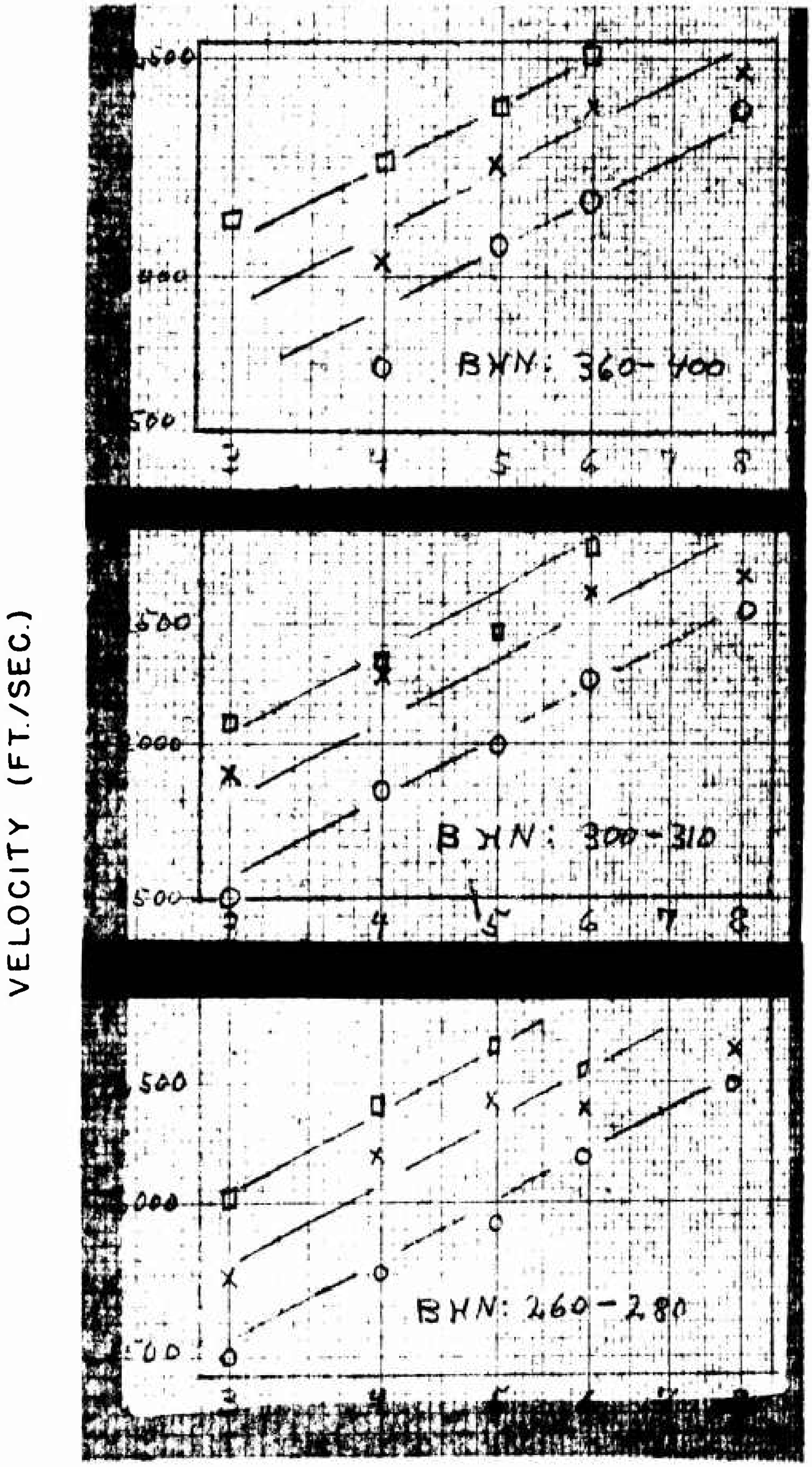
EFFECT OF OBLIQUITY UPON
ARMY BALLISTIC LIMIT
(AVERAGED OVER VARIOUS PLATE THICKNESSES)

FIG. 4



HARDNESS DEPENDENCE OF
BALLISTIC LIMIT COEFFICIENTS

FIG. 5



□ 30° OBLIQUITY
× 20° "
⊙ 0° "

PLATE THICKNESS (IN $\frac{1}{8}$")

FIG. 6

NAVY BALLISTIC LIMITS

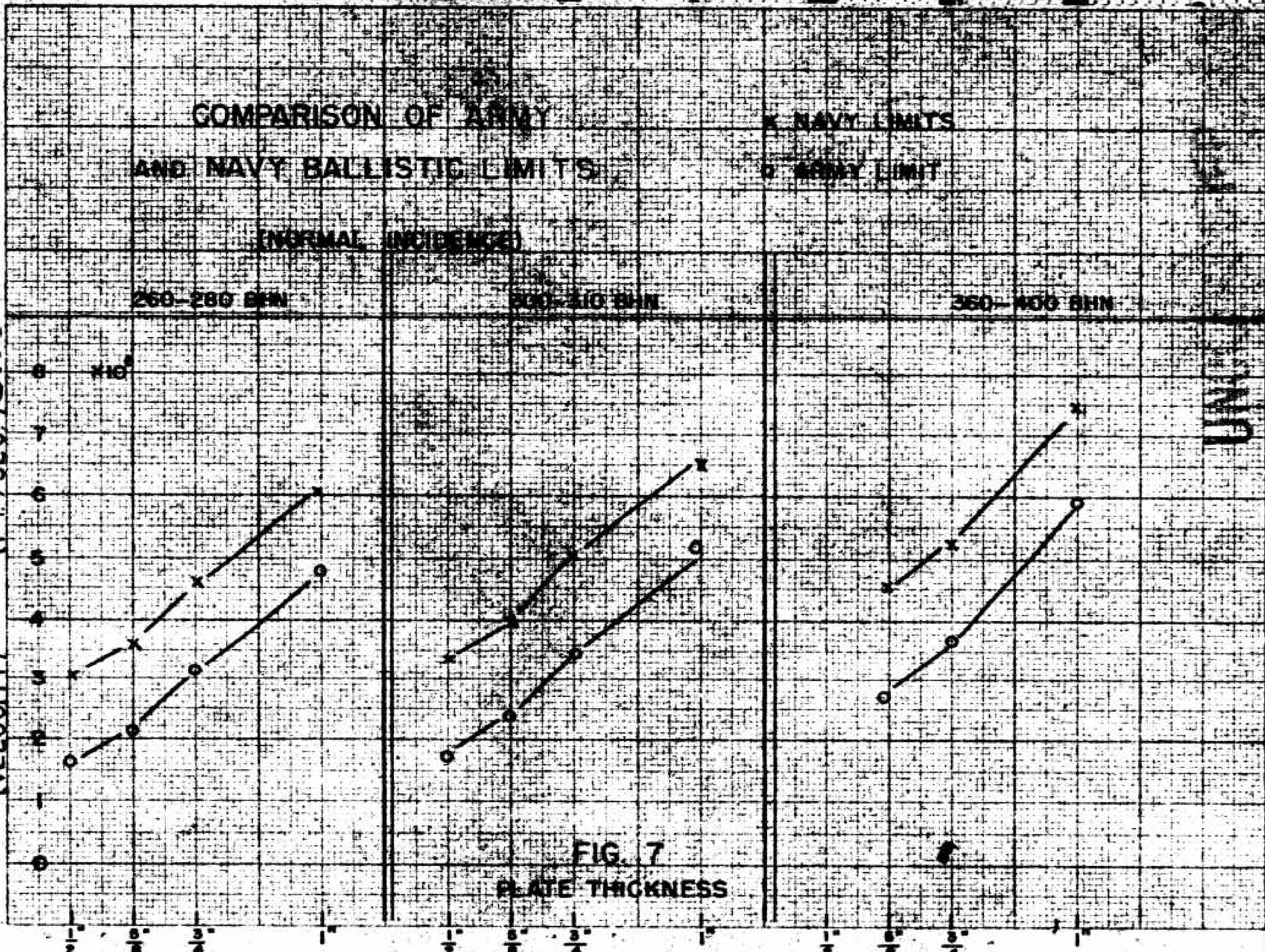

COMPARISON OF ARMY
AND NAVY BALLISTIC LIMITS
(NORMAL INCIDENCE)
x NAVY LIMITS
o ARMY LIMIT
260-280 BHN
300-310 BHN
360-400 BHN
(VELOCITY)^2 (FT.^{2}/SEC.^{2})
$\times 10^6$
0
1
2
3
4
5
6
7
8
$\frac{1}{2}$"
$\frac{5}{8}$"
$\frac{3}{4}$"
1"
PLATE THICKNESS

FIG. 7